

All-angle zero reflection at metamaterial surfaces

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The authors study theoretically reflection on the surface of a metamaterial with a hyperbolic dispersion. It is found that reflection is strongly dependent on how the surface is terminated with respect to the asymptote of the hyperbolic dispersion. For a surface terminated normally to the asymptote, zero reflection occurs for all incident angles. It is exemplified by a metamaterial made of a periodic metal-dielectric layered structure with its surface properly cut through numerical simulations.

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Metamaterials are artificially designed composites consisting of periodic subwavelength structures. The optical response of metamaterials originates from their structures instead from their compositions, leading to many unusual optical properties that do not occur in nature. For instance, metamaterials with a negative refractive index can produce negative refraction^{1,2,3} and superlensing.^{4,5,6} In contrast to conventional materials, the energy transport through negative-refractive-index metamaterials is in a direction opposite to the phase direction, giving rise to reversed Doppler effects and inverted Cherenkov cone.^{7,8} Metamaterials can also be used to construct invisible cloak.^{9,10}

Reflection is a wave phenomenon occurring for waves impinging upon a surface. Our common understanding is that reflection is inevitable although it can be eliminated at some special incident angles, e.g., Brewster's angles. In this Letter, we show theoretically that all-angle zero reflection can occur at the surface of a metamaterial with a hyperbolic dispersion. It is exemplified by a metamaterial made of a periodic metal-dielectric layered structure through numerical simulations.

For any reflection on a surface, the spatial and time variation of all fields must be the same at the surface. As a result, the in-plane wave vector of an incident wave should be equal to that of the reflected one, independent of the nature of the boundary conditions.¹¹ For homogeneous media, this leads to the fact that the incident angle is equal to the reflected angle. For anisotropic media, however, incident and reflected angles may not be the same. For either isotropic media or anisotropic media with an elliptic dispersion, reflection always exists due to the fact that the conservation of in-plane wave vectors of incident and reflected waves can be always satisfied, regardless of the cutting directions of surfaces, as shown schematically in Fig. 1. For an anisotropic medium

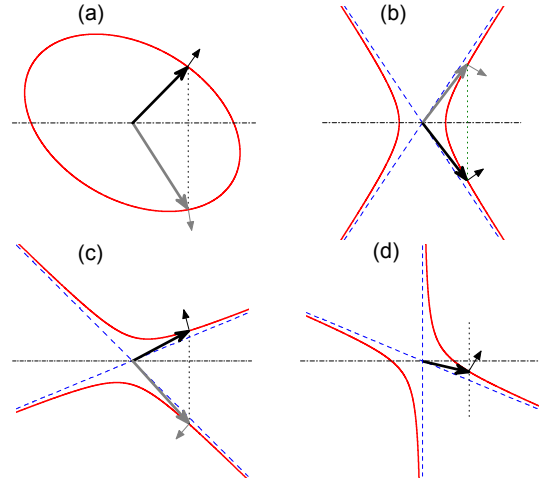


FIG. 1: (Color online) Equi-frequency surface analysis of reflection on surfaces of metamaterials with (a) an elliptic and (b-d) a hyperbolic dispersion. Surfaces (dash-dotted lines) lie in the horizontal plane. For metamaterials with the hyperbolic dispersion, surfaces are cut along the principal axis (b), perpendicularly to the asymptote (dashed lines) of the hyperbolic dispersion (d), and obliquely to both the principal axis and asymptote (c). Black (grey) thick arrows denote the incident (reflected) wave vector. Thin arrows indicate the direction of the group velocity. Dotted lines illustrate the conservation of the in-plane wave vectors. Note that metamaterials occupy the half-space below the horizontal line and waves are incident from the metamaterial side.

with a hyperbolic dispersion, reflection is sensitive to the surface termination direction, namely, the surface orientation with respect to the asymptote of the hyperbolic dispersion. For surfaces terminated along the principal axes, reflection always exists with equal incident and reflected angles. For surfaces oriented obliquely to both the asymptotes and principal axes, the reflected wave vector still exists with the reflected angle different from the incident angle. If the surface is cut perpendicularly to the asymptote, however, we cannot find reflected wave vec-

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tor for any given incident wave vector. In this situation, zero reflection is expected for all incident angles as to be shown later. It should be noted that the energy flow direction is the same as that of the group velocity, defined by $\mathbf{v}_g = \nabla\omega(\mathbf{k})$, where k is the wave vector and $\omega(\mathbf{k})$ is the dispersion relation. From its definition, the group velocity direction is perpendicular to the equi-frequency surface and points to the direction along which $\omega(\mathbf{k})$ is increasing.

A qualitative analysis of all-angle zero reflection was given hereinbefore. In the following, we would like to give a rigorous proof. We consider an anisotropic metamaterial whose permittivity tensor and permeability tensor are both diagonalizable.¹² To simply the proceeding analysis, we assume that the metamaterial is nonmagnetic, namely, the diagonal elements of the permeability tensor are all equal to 1. Without loss of generality, we assume in our analysis a plane wave with the magnetic field polarized along the y direction with the form $\mathbf{H} = H_0 \hat{\mathbf{y}} \exp[i(k_x x + k_z z) - \omega t]$, where $\hat{\mathbf{y}}$ is the unit vector along the y direction. If we choose a Cartesian coordinate system x - z with its axes along the principal axes of the metamaterial, this plane wave satisfies the following dispersion relation

$$\frac{k_x^2}{\varepsilon_x} + \frac{k_z^2}{\varepsilon_z} = \frac{\omega^2}{c^2}, \quad (1)$$

where ε_x and ε_z are the diagonal elements of the permittivity tensor. For ε_x and ε_z with opposite signs, the corresponding dispersion is hyperbolic. If we choose a new Cartesian coordinate system x' - z' with the same origin, the permittivity tensor is no longer diagonal and is transformed to

$$\begin{bmatrix} \varepsilon_{x'x'} & \varepsilon_{x'z'} \\ \varepsilon_{z'x'} & \varepsilon_{z'z'} \end{bmatrix} = \begin{bmatrix} c^2 \varepsilon_x + s^2 \varepsilon_z & cs(\varepsilon_x - \varepsilon_z) \\ cs(\varepsilon_x - \varepsilon_z) & s^2 \varepsilon_x + c^2 \varepsilon_z \end{bmatrix}. \quad (2)$$

Here, the parameters c and s are given by

$$c = \cos^2 \theta, \quad s = \sin^2 \theta, \quad (3)$$

where θ is the angle between the x' and x axes. The dispersion in the new coordinate system becomes accordingly

$$\frac{\varepsilon_{x'x'}^2 k_{x'}^2 + \varepsilon_{z'z'}^2 k_{z'}^2 + 2\varepsilon_{x'z'} k_{x'} k_{z'}}{\varepsilon_{x'x'} \varepsilon_{z'z'} - \varepsilon_{x'z'}^2} = \frac{\omega^2}{c^2}. \quad (4)$$

For a metamaterial with a hyperbolic dispersion, reflection is strongly dependent on the cutting direction of the surface. Without loss of generality, we assume that the metamaterial surface is terminated normally to the z' axis, i.e., in the $x'y'$ plane. Consequently, $k_{x'}$ is the in-plane wave vector of an plane wave and $k_{z'}$ is the perpendicular component. For a given in-plane wave vector $k_{x'}$ of an incident plane wave, we can always find a real solution of $k_{z'}$ for the reflected wave from the conservation of the in-plane wave vector, if the x' axis is not perpendicular to the asymptote of the hyperbolic dispersion. For the x' axis just perpendicular to the asymptote,

no real solution of $k_{z'}$ for the reflected wave can be found. In this case, $k_{z'}$ should be a complex number possessing both a real and an imaginary part. Zero reflection is expected as can be confirmed by calculating the normal component of the reflected Poynting vector with respect to the surface, defined in cgs units by

$$S_{r\perp} = \frac{4\pi}{c} \text{Re}(\mathbf{E}_r \times \mathbf{H}_r^*)_{\perp} = \frac{4\pi}{c} \text{Re}(E_{rx'} H_{ry'}^*), \quad (5)$$

where E_r and H_r are the reflected electric and magnetic fields with their components related each other by

$$E_{rx'} = -\frac{c}{\omega} \frac{\varepsilon_{z'z'} k_{z'} + \varepsilon_{x'z'} k_{x'}}{\varepsilon_{x'x'} \varepsilon_{z'z'} - \varepsilon_{x'z'}^2} H_{ry'}. \quad (6)$$

Suppose a complex $k_{z'}$ for the reflected wave and substitute it into Eq. (4) we can obtain

$$\varepsilon_{z'z'} \text{Re}(k_{z'}) + \varepsilon_{x'z'} k_{x'} = 0. \quad (7)$$

This immediately leads to the fact that $E_{rx'}$ does not possess a real part, leading to $S_{r\perp} = 0$. We can thus conclude that the reflected wave does not carry any energy along the surface normal, implying zero reflection.

One feasible realization of a metamaterial with a hyperbolic dispersion is to adopt a periodic metal-dielectric layered structure.^{13,14,15,16} In the long wavelength limit (the period is much smaller than the operating wavelength), this periodic metal-dielectric layered structure can be viewed as a effective anisotropic metamaterial with the permittivity tensor given by

$$\begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_x & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}. \quad (8)$$

For p -polarized waves, ε_x and ε_z are related to the parameters of the constituents by^{13,17}

$$\varepsilon_x = \frac{\varepsilon_1 d_1 + \varepsilon_2 d_2}{d}, \quad (9a)$$

$$\varepsilon_z = \frac{\varepsilon_1 \varepsilon_2 d}{\varepsilon_1 d_2 + \varepsilon_2 d_1}, \quad (9b)$$

where $\varepsilon_{1,2}$ is the dielectric constant of the constituents, $d_{1,2}$ is the thickness, and $d = d_1 + d_2$ is the period. If we adopt a Drude model¹⁸ to describe the dielectric constant of the metal constituent, namely, $\varepsilon(\omega) = 1 - \omega_p^2/\omega^2$, where ω_p is the plasma frequency of the metal, it can be easily shown that ε_x and ε_z can have opposite signs at certain frequencies for a proper choice of the thickness parameters. This leads to a hyperbolic dispersion for the periodic metal-dielectric layered structure.

To illustrate all-angle zero reflection on the surface of a periodic metal-dielectric layered structure, we carry out finite-difference time-domain (FDTD) simulations¹⁹ with perfectly matched layer boundary conditions,²⁰ shown in Fig. 2. Without loss of generality, the periodic metal-dielectric layered structure is assumed to made from Ag

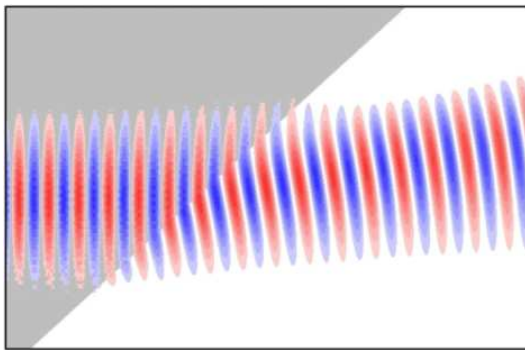


FIG. 2: (Color online) FDTD simulations of the magnetic field distributions for a p -polarized Gaussian beam launched from a periodic metal-dielectric layered structure upon an interface between the structure (grey area) and a dielectric medium (white area) with a dielectric constant of 2.5.

and a dielectric with a dielectric constant of 2.231. The thickness of the Ag layer is 10 nm and that of the dielectric layer is 50 nm. In our simulations, a p -polarized Gaussian beam is used. Its wavelength is 756 nm, which is much larger than the period (60 nm) of the periodic metal-dielectric layered structure, justifying our effective medium approximation. An experimental value²¹ of the refractive index $n = 0.03 + i5.242$ at 756 nm for Ag is used. These parameters result in $\text{Re}(\epsilon_x) = 2.72$ and $\text{Re}(\epsilon_z) = -2.72$, leading to a rectangularly hyperbolic dispersion for the periodic metal-dielectric layered structure.

The terminated surface of the periodic metal-dielectric layered structure is perpendicular to one of the asymp-

tot of the hyperbolic dispersion, namely, in an angle of 45° with respect to the periodic direction. The incident Gaussian beam forms an angle of 45° with respect to the surface normal and is perpendicular to the periodic direction. It is obvious from the FDTD simulations that no reflection occurs at the surface. It should be noted that zero reflection does not depend on the incident angle. Simulations for other incident angles are also conducted and zero reflection is always found, manifesting all-angle zero reflection.

In conclusion, the reflection on the surface of a metamaterial with a hyperbolic dispersion is studied theoretically. For the metamaterial surface terminated obliquely to the asymptote of the hyperbolic dispersion, reflection is inevitable. However, all-angle zero reflection can occur if the surface is cut perpendicularly to the asymptote. We show this by a rigorous proof that for any incident angle the surface normal component of the Poynting vector of the reflected wave does not possess a real part, implying zero energy flow along the surface normal. Numerical simulations of a periodic metal-dielectric layered structure that is cut properly affirm unambiguously our theoretical prediction of all-angle zero reflection. Metamaterials with all-angle zero reflection at their surfaces could be exploited in many applications to eliminate undesirable reflection.

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